A nucleon in a tiny box

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Lattice QCD simulations are necessarily performed in finite boxes. Finite-size effects are controlled by the parameters m L, where L is the lattice size and m the mass of the lightest particle, namely the pion in QCD. Physical results can be obtained in the limit m L >> 1. As the pion masses achieved in simulations approach the physical value it becomes harder to fulfill this condition. However, most of the configurations in large lattices describe pions traveling at large distances of the order of L. Since the physics of these soft-pions is strongly constrained by chiral symmetry, strong theoretical control over them makes their numerical simulation is unnecessary. One can thus obtain physical results by simulating in smaller lattices and using Chiral Perturbation Theory (CPT) or some other relevant effective theory to include the soft-pion physics cut off by the box size and extrapolate the results to the infinite volume limit. Another way to describe the same procedure gives added insight: The low-energy physics in the infinite and finite volume are described by the same effective theory with the same low-energy constants, since the values of these constants encapsulate short-distance physics that is not modified by finite-volume effects. The comparison of finite volume lattice results with the effective theory prediction allows one therefore to determine the value of some of the low energy constants. Those, in turn, can be used to determine physical observables in the infinite-volume limit.

This general procedure has been carried out in the regime m L >> 1, where standard CPTs can be applied, to a variety of one nucleon observables. However, it is for m L << 1 (in the so-called epsilon-regime that the programme described above is fully realized. For such small boxes, most of the pion cloud surrounding a baryon is excluded, and we are left with a bare nucleon. There are some modifications to the usual CPTs power counting in this regime. The first and obvious one is that the momenta are quantized in units of $2\pi/L$. More importantly, the pion zero mode fluctuations are not suppressed, become non-perturbative and need to be treated exactly [1]. They reduce the value of the chiral condensate and make the chiral condensate disappear altogether in the chiral limit. This is to be expected since there is no chiral symmetry breaking at finite volumes. Recently, the epsilon-regime in the meson sector and its relevance to lattice QCD have been assessed in a number of papers. In the present work, we consider the one-baryon sector. Convergence in the baryonic sector is typically worse than in the mesonic sector, as it receives contributions at every order in $p/(4\pi f)$, unlike the meson sector case where the expansion parameter is $(p/(4\pi f))^2$. We address this issue by comparing the sizes of leading and next-to-leading order contributions in a calculation of the nucleon mass.

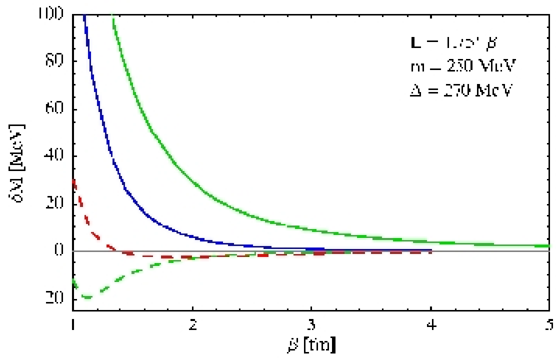

FIG. 1: Examples of the absolute value of the finite volume mass-shift of the nucleon at leading order (solid lines) and the next-to-leading order *correction* (dashed lines) in MeV as function of L [fm] for L=1.75 β. The lighter (green) lines show the result with (m=Δ=0), the darker lines include finite m and Δ. The parameters are m=250 MeV, Δ=270 MeV.

REFERENCES

[1] J. Gasser and H. Leutwyler, Phys. Lett. B 188, 477 (1987).